Write your homework *neatly*, *in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. If the problem involves a function, write the function. If the problem involves an equation, write the equation. Use words, and when appropriate, *write in sentences*.

An interval is *maximal* with respect to a condition if it is not a proper subset of another interval which satisfies the condition. That is, if it is not contained in a bigger interval which also satisfies the condition.

An *interior point* of an interval is a point in the interval which is not an endpoint.

The phrase "find and classify the critical points of a function f" means:

- Find all interior points x in the domain of f such that $f'(x) = 0$ or $f'(x)$ does not exist.

- Determine whether each critical point gives a local maximum, a local minimum, or neither.

Problem 1. Let
$$f(x) = 3x^5 - 4x^3 - 3x.$$
Find and classify the critical points of f.

Fact 1. Recall the quadratic formula: if $f(x) = ax^2 + bx + c = 0$, then
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The *discriminant* of f is
$$\Delta = b^2 - 4ac.$$

Then

- if $\Delta > 0$, f has exactly two real zeros.

- if $\Delta = 0$, then f has exactly one real zero.

- if $\Delta < 0$, then f has no real zeros.

Use this basic fact to solve the following problem.

Problem 2. Consider the cubic polynomial
$$f(x) = x^3 + ax^2 + bx.$$
Since f is a polynomial of odd degree, f has at least one real zero.

 (a) Find the values of a and b for which f has exactly three zeros.

 (b) Find the values of a and b for which f has exactly two zeros.

 (c) Find the values of a and b for which f has exactly one zero.

 (d) Find the values of a and b for which f has exactly two local extrema.

 (e) Find the values of a and b for which f has exactly one horizontal tangent.

 (f) Find the values of a and b for which f has no horizontal tangents.

Problem 3. Let
$$f(x) = 3x^4 - 16x^3 + 24x^2 + 48.$$

 (a) Find f''.

 (b) Solve $f''(x) = 0$ and create a sign chart for f''.

 (c) Identity maximal intervals on which f is concave up or concave down.

Problem 4. Thomas Problem §4.5 # 4.

Problem 5. Thomas Problem §4.5 # 7.